

Appendix A

Microscopic transport equations

A.1 Rectangular coordinates

Mass balance for component A (ρ and D_{AB} constant)

$$\frac{\partial C_A}{\partial t} + v_x \frac{\partial C_A}{\partial x} + v_y \frac{\partial C_A}{\partial y} + v_z \frac{\partial C_A}{\partial z} = D_{AB} \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} \right) + R_A.$$

Total mass balance (ρ constant)

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0.$$

Energy balance (ρ and k constant)

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S.$$

Momentum balance (ρ and μ constant)

$$\begin{aligned} x : \quad \rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) &= -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) \\ &\quad + \rho g_x, \\ y : \quad \rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) &= -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) \\ &\quad + \rho g_y, \\ z : \quad \rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) \\ &\quad + \rho g_z. \end{aligned}$$

A.2 Cylindrical coordinates (mass and energy)

Mass balance for component A (ρ and D_{AB} constant)

$$\frac{\partial C_A}{\partial t} + v_r \frac{\partial C_A}{\partial r} + v_\theta \frac{1}{r} \frac{\partial C_A}{\partial \theta} + v_z \frac{\partial C_A}{\partial z} = D_{AB} \left(\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial C_A}{\partial r} + \frac{1}{r^2} \frac{\partial^2 C_A}{\partial \theta^2} + \frac{\partial^2 C_A}{\partial z^2} \right) + R_A.$$

Energy balance (ρ and k constant)

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_\theta \frac{1}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S.$$

A.3 Spherical coordinates (mass and energy)

Mass balance for component A (ρ and D_{AB} constant)

$$\begin{aligned} \frac{\partial C_A}{\partial t} + v_r \frac{\partial C_A}{\partial r} + v_\theta \frac{1}{r} \frac{\partial C_A}{\partial \theta} + v_\phi \frac{1}{r \sin \theta} \frac{\partial C_A}{\partial \phi} \\ = D_{AB} \left(\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial C_A}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial C_A}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 C_A}{\partial \phi^2} \right) + R_A. \end{aligned}$$

Energy balance (ρ and k constant)

$$\begin{aligned} \rho c_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_\theta \frac{1}{r} \frac{\partial T}{\partial \theta} + v_\phi \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) \\ = k \left(\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial T}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial T}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S. \end{aligned}$$

Appendix B

Dimensionless variables

Useful dimensionless numbers can be created by taking the ratios of different forces, mass fluxes, or heat fluxes. The dimensionless number will indicate which of the forces or fluxes is the most important. Various dimensionless quantities are given in Tables B.1–B.4.

Table B.1. Dimensionless numbers based on force

	Inertial	Viscous	Buoyancy	Surface tension	Pressure
Force (kg m s ⁻²)	$\rho U^2 L^2$	μUL	$gL^3 \Delta\rho$	γL	$L^2 \Delta p$
Inertial	–	$Re = \frac{\rho UL}{\mu}$	$Fr = \frac{gL \Delta\rho}{\rho U^2}$	$We = \frac{\rho U^2 L}{\gamma}$	$Eu = \frac{\Delta p}{\rho U^2}$
Viscous		–	$\frac{gL^2 \Delta\rho}{\mu U}$	$Ca = \frac{\mu U}{\gamma}$	$\frac{L \Delta p}{\mu U}$
Buoyancy			–	$Eo = \frac{gL^2 \Delta\rho}{\gamma}$	$\frac{\Delta p}{gL \Delta\rho}$
Surface tension				–	$\frac{L \Delta p}{\gamma}$
Pressure					–

Table B.2. Dimensionless numbers based on mass flux

	Convective	Diffusion	Interface	Reaction
Mass flux (mol s ⁻¹)	$U \Delta C L^2$	$D \Delta C L$	$k_w \Delta C L^2$	$r_A L^3$
Convective	–	$Pe = \frac{UL}{D}$	$St = \frac{k_w}{U}$	$Da_I = \frac{Lr_A}{U \Delta C}$
Diffusive		–	$Sh = \frac{k_w L}{D}$	$Da_{II} = \frac{L^2 r_A}{D \Delta C}$
Interface			–	$\frac{r_A L}{k_w \Delta C}$
Reaction				–

Table B.3. Dimensionless numbers based on heat flux

	Convective	Conductive	Interphase	Reaction
Heat flux (J s^{-1})	$\rho C_p \Delta T U L^2$	$k \Delta T L$	$h \Delta T L^2$	$r_A L^3 \Delta H$
Convective	–	$Pe = \frac{\rho c_p U L}{k}$	$St = \frac{h}{c_p \rho U}$	$Da_{III} = \frac{r_A L \Delta H}{\rho c_p U \Delta T}$
Conductive		–	$Nu = \frac{h L}{k}$	$Da_{IV} = \frac{r_A \Delta H L^2}{k \Delta T}$
Interphase			–	$\frac{r_A L \Delta H}{h \Delta T}$
Reaction				–

Table B.4. Common dimensionless numbers

Name		Definition
Biot	$Bi = \frac{hL}{k}$	$\frac{\text{external heat transfer}}{\text{internal conductive heat transfer}}$
Capillary	$Ca = \frac{\mu U}{\gamma}$	$\frac{\text{viscous force}}{\text{capillary force}}$
Damköhler I	$Da = \frac{k_n C_0^n}{C_0 U / L} = k_n C_0^{n-1} \tau$	$\frac{\text{reaction rate}}{\text{convective mass transport rate}}$
Damköhler II	$Da_{II} = \frac{k_n C_0^{n-1}}{k_g a}$	$\frac{\text{reaction rate}}{\text{mass transport rate}}$
Damköhler III	$Da_{III} = \frac{k_n C_0^{n-1} L \Delta H}{\rho c_p U \Delta T}$	$\frac{\text{heat of reaction}}{\text{heat transport rate}}$
Eotvos	$Eu = \frac{gL^2 \Delta \rho}{\gamma}$	$\frac{\text{buoyancy force}}{\text{surface tension force}}$
Euler	$Eu = \frac{\Delta p}{\rho U^2}$	$\frac{\text{pressure force}}{\text{inertia force}}$
Fourier	$Fo = \frac{\alpha t}{L^2}$	$\frac{\text{heat conduction}}{\text{heat accumulation}}$
Froude	$Fr = \frac{gL \Delta \rho}{\rho U^2}$	$\frac{\text{buoyed weight}}{\text{surface inertia force}}$
Grashof	$Gr = \frac{\rho^2 g \beta (T_s - T_\infty) L^3}{\mu^2}$	$\frac{(\text{buoyancy}) (\text{inertia force})}{\text{viscous force}}$
Nusselt	$Nu = \frac{hL}{k}$	$\frac{\text{convective heat transfer}}{\text{conductive heat transfer}}$
Péclet	$Pe_H = \frac{LU}{\alpha}$ or $Pe_D = \frac{LU}{D}$	$\frac{\text{convective transport}}{\text{diffusive transport}}$
Prandtl	$Pr = \frac{\mu}{\rho \alpha}$	$\frac{\text{viscosity}}{\text{heat diffusivity}}$
Rayleigh	$Ra = \frac{\rho g \beta (T_s - T_\infty) L^3}{\mu \alpha}$	$\frac{\text{free convection}}{\text{conduction}}$

(cont.)

Table B.4. (cont.)

Name		Definition
Reynolds	$Re = \frac{\rho UL}{\mu}$	$\frac{\text{inertia force}}{\text{viscous force}}$
Schmidt	$Sc = \frac{\mu}{\rho D}$	$\frac{\text{viscosity}}{\text{diffusivity}}$
Sherwood	$Sh = \frac{k_c L}{D}$	$\frac{\text{convective mass transfer}}{\text{conductive mass transfer}}$
Stokes	$St = \frac{\tau_p}{\tau_{fluid}}$	$\frac{\text{particle relaxation time}}{\text{fluid characteristic time}}$
Weber	$We = \frac{\rho U^2 L}{\gamma}$	$\frac{\text{inertial force}}{\text{surface tension force}}$

Appendix C

Student's t-distribution

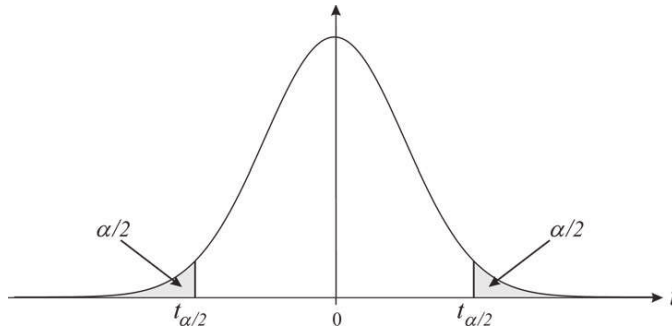


Table C.1. t-table

Degrees of freedom	Probability (Area in one tail, $\frac{\alpha}{2}$)						
	$t_{0.100}$	$t_{0.05}$	$t_{0.025}$	$t_{0.0100}$	$t_{0.0050}$	$t_{0.001}$	$t_{0.0005}$
1	3.08	6.31	12.71	31.82	63.66	318.31	636.62
2	1.89	2.92	4.30	6.96	9.92	22.33	31.60
3	1.64	2.35	3.18	4.54	5.84	10.21	12.92
4	1.53	2.13	2.78	3.75	4.60	7.17	8.61
5	1.48	2.02	2.57	3.36	4.03	5.89	6.87
6	1.44	1.94	2.45	3.14	3.71	5.21	5.96
7	1.41	1.89	2.36	3.00	3.50	4.79	5.41
8	1.40	1.86	2.31	2.90	3.36	4.50	5.04
9	1.38	1.83	2.26	2.82	3.25	4.30	4.78
10	1.37	1.81	2.23	2.76	3.17	4.14	4.59
11	1.36	1.80	2.20	2.72	3.11	4.02	4.44
12	1.36	1.78	2.18	2.68	3.05	3.93	4.32
13	1.35	1.77	2.16	2.65	3.01	3.85	4.22
14	1.34	1.76	2.14	2.62	2.98	3.79	4.14
15	1.34	1.75	2.13	2.60	2.95	3.73	4.07
16	1.34	1.75	2.12	2.58	2.92	3.69	4.01
17	1.33	1.74	2.11	2.57	2.90	3.65	3.97
18	1.33	1.73	2.10	2.55	2.88	3.61	3.92
19	1.33	1.73	2.09	2.54	2.86	3.58	3.88
20	1.33	1.72	2.09	2.53	2.85	3.55	3.85
21	1.32	1.72	2.08	2.52	2.83	3.53	3.82
22	1.32	1.72	2.07	2.51	2.82	3.50	3.79
23	1.32	1.71	2.07	2.50	2.81	3.48	3.77
24	1.32	1.71	2.06	2.49	2.80	3.47	3.75
25	1.32	1.71	2.06	2.49	2.79	3.45	3.73
26	1.32	1.71	2.06	2.48	2.78	3.44	3.71
27	1.31	1.70	2.05	2.47	2.77	3.42	3.69
28	1.31	1.70	2.05	2.47	2.76	3.41	3.67
29	1.31	1.70	2.05	2.46	2.76	3.40	3.66
30	1.31	1.70	2.04	2.46	2.75	3.39	3.65
40	1.30	1.68	2.02	2.42	2.70	3.31	3.55
50	1.30	1.68	2.01	2.40	2.68	3.26	3.50
70	1.29	1.67	1.99	2.38	2.65	3.21	3.44
90	1.29	1.66	1.99	2.37	2.63	3.18	3.40
100	1.29	1.66	1.98	2.36	2.63	3.17	3.39
150	1.29	1.66	1.98	2.35	2.61	3.15	3.36
∞	1.28	1.64	1.96	2.33	2.58	3.09	3.29

C.1 The F-distribution

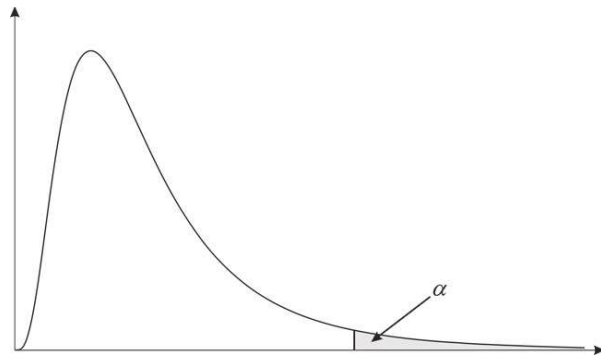
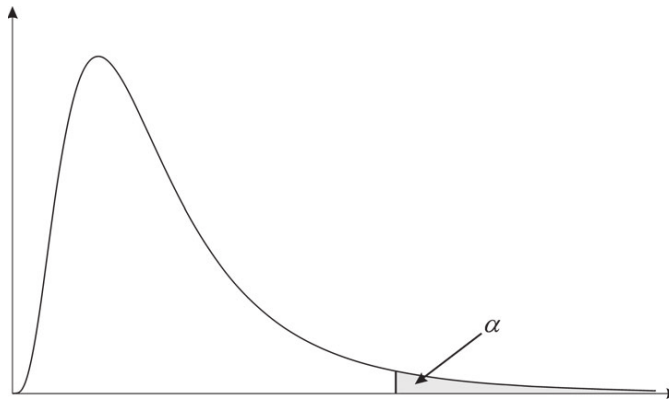


Table C.2. F-table; 10% confidence limit

$v_1 \backslash v_2$ Degrees of freedom	1	2	3	4	5	6	7	8	9	10
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.20
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42
10	3.28	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71
80	2.77	2.37	2.15	2.02	1.92	1.85	1.79	1.75	1.71	1.68
100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69	1.66
150	2.74	2.34	2.12	1.98	1.89	1.81	1.76	1.71	1.67	1.64
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60



12	15	20	25	30	40	60	100	150	∞
60.71	61.22	61.74	62.05	62.26	62.53	62.79	63.01	63.11	63.33
9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.48	9.49
5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.14	5.14	5.13
3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.77	3.76
3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.13	3.12	3.10
2.90	2.87	2.84	2.81	2.80	2.78	2.76	2.75	2.74	2.72
2.67	2.63	2.59	2.57	2.56	2.54	2.51	2.50	2.49	2.47
2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.31	2.29
2.38	2.34	2.30	2.27	2.25	2.23	2.21	2.19	2.18	2.16
2.28	2.24	2.20	2.17	2.16	2.13	2.11	2.09	2.08	2.06
2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.99	1.97
2.15	2.10	2.06	2.03	2.01	1.99	1.96	1.94	1.93	1.90
2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.87	1.85
2.05	2.01	1.96	1.93	1.91	1.89	1.86	1.83	1.82	1.80
2.02	1.97	1.92	1.89	1.87	1.85	1.82	1.79	1.78	1.76
1.99	1.94	1.89	1.86	1.84	1.81	1.78	1.76	1.74	1.72
1.96	1.91	1.86	1.83	1.81	1.78	1.75	1.73	1.71	1.69
1.93	1.89	1.84	1.80	1.78	1.75	1.72	1.70	1.68	1.66
1.91	1.86	1.81	1.78	1.76	1.73	1.70	1.67	1.66	1.63
1.89	1.84	1.79	1.76	1.74	1.71	1.68	1.65	1.64	1.61
1.88	1.83	1.78	1.74	1.72	1.69	1.66	1.63	1.62	1.59
1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.61	1.60	1.57
1.84	1.80	1.74	1.71	1.69	1.66	1.62	1.59	1.58	1.55
1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.58	1.56	1.53
1.82	1.77	1.72	1.68	1.66	1.63	1.59	1.56	1.55	1.52
1.81	1.76	1.71	1.67	1.65	1.61	1.58	1.55	1.54	1.50
1.80	1.75	1.70	1.66	1.64	1.60	1.57	1.54	1.52	1.49
1.79	1.74	1.69	1.65	1.63	1.59	1.56	1.53	1.51	1.48
1.78	1.73	1.68	1.64	1.62	1.58	1.55	1.52	1.50	1.47
1.77	1.72	1.67	1.63	1.61	1.57	1.54	1.51	1.49	1.46
1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.43	1.42	1.38
1.66	1.60	1.54	1.50	1.48	1.44	1.40	1.36	1.34	1.29
1.63	1.57	1.51	1.47	1.44	1.40	1.36	1.32	1.30	1.24
1.61	1.56	1.49	1.45	1.42	1.38	1.34	1.29	1.27	1.21
1.59	1.53	1.47	1.43	1.40	1.35	1.30	1.26	1.23	1.17
1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.18	1.15	1.00

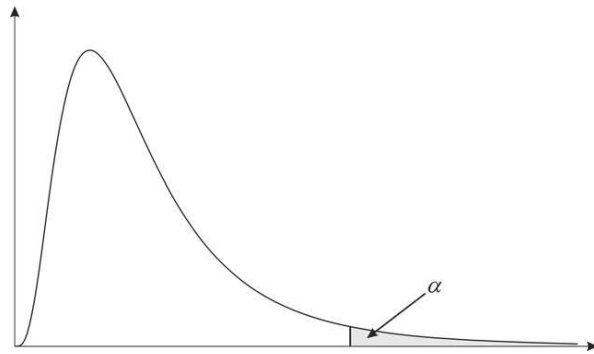
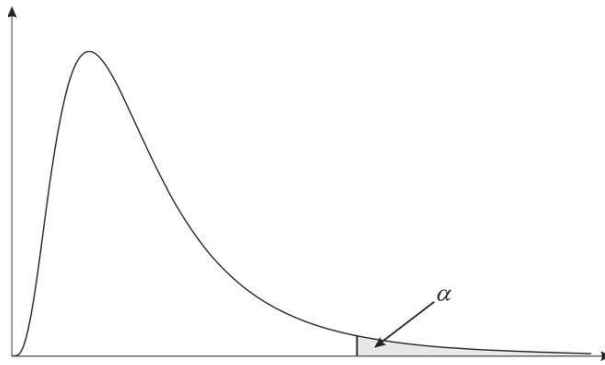


Table C.3. F-table; 5% confidence limit

$V_1 \backslash V_2$ Degrees of freedom	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83



	12	15	20	25	30	40	60	100	150	∞
243.91	245.95	248.01	249.26	250.10	251.14	252.20	253.04	253.46	254.31	
19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.49	19.49	19.50	
8.74	8.70	8.66	8.63	8.62	8.59	8.57	8.55	8.54	8.53	
5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.65	5.63	
4.68	4.62	4.56	4.52	4.50	4.46	4.43	4.41	4.39	4.36	
4.00	3.94	3.87	3.83	3.81	3.77	3.74	3.71	3.70	3.67	
3.57	3.51	3.44	3.40	3.38	3.34	3.30	3.27	3.26	3.23	
3.28	3.22	3.15	3.11	3.08	3.04	3.01	2.97	2.96	2.93	
3.07	3.01	2.94	2.89	2.86	2.83	2.79	2.76	2.74	2.71	
2.91	2.84	2.77	2.73	2.70	2.66	2.62	2.59	2.57	2.54	
2.79	2.72	2.65	2.60	2.57	2.53	2.49	2.46	2.44	2.40	
2.69	2.62	2.54	2.50	2.47	2.43	2.38	2.35	2.33	2.30	
2.60	2.53	2.46	2.41	2.38	2.34	2.30	2.26	2.24	2.21	
2.53	2.46	2.39	2.34	2.31	2.27	2.22	2.19	2.17	2.13	
2.48	2.40	2.33	2.28	2.25	2.20	2.16	2.12	2.10	2.07	
2.42	2.35	2.28	2.23	2.19	2.15	2.11	2.07	2.05	2.01	
2.38	2.31	2.23	2.18	2.15	2.10	2.06	2.02	2.00	1.96	
2.34	2.27	2.19	2.14	2.11	2.06	2.02	1.98	1.96	1.92	
2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.94	1.92	1.88	
2.28	2.20	2.12	2.07	2.04	1.99	1.95	1.91	1.89	1.84	
2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.88	1.86	1.81	
2.23	2.15	2.07	2.02	1.98	1.94	1.89	1.85	1.83	1.78	
2.20	2.13	2.05	2.00	1.96	1.91	1.86	1.82	1.80	1.76	
2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.80	1.78	1.73	
2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.78	1.76	1.71	
2.15	2.07	1.99	1.94	1.90	1.85	1.80	1.76	1.74	1.69	
2.13	2.06	1.97	1.92	1.88	1.84	1.79	1.74	1.72	1.67	
2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.73	1.70	1.65	
2.10	2.03	1.94	1.89	1.85	1.81	1.75	1.71	1.69	1.64	
2.09	2.01	1.93	1.88	1.84	1.79	1.74	1.70	1.67	1.62	
2.00	1.92	1.84	1.78	1.74	1.69	1.64	1.59	1.56	1.51	
1.92	1.84	1.75	1.69	1.65	1.59	1.53	1.48	1.45	1.39	
1.88	1.79	1.70	1.64	1.60	1.54	1.48	1.43	1.39	1.32	
1.85	1.77	1.68	1.62	1.57	1.52	1.45	1.39	1.36	1.28	
1.82	1.73	1.64	1.58	1.54	1.48	1.41	1.34	1.31	1.22	
1.75	1.67	1.57	1.51	1.46	1.39	1.32	1.24	1.20	1.00	

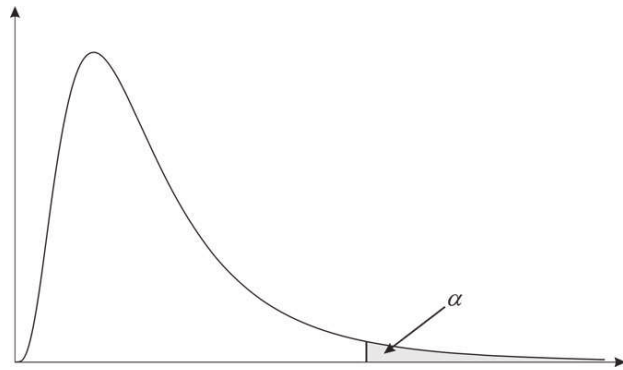
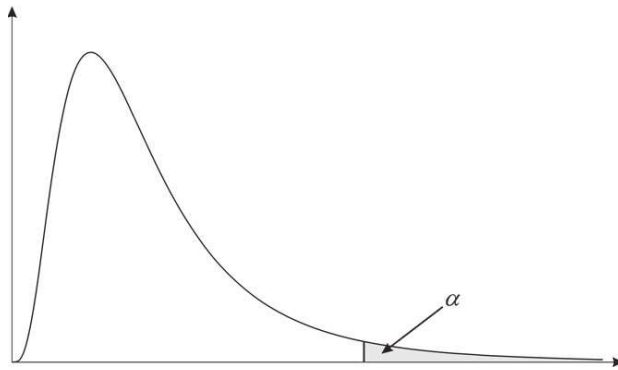


Table C.4. F-table; 1% confidence limit

$v_1 \backslash v_2$ Degrees of freedom	1	2	3	4	5	6	7	8	9	10
1	4.052	5.000	5.403	5.625	5.764	5.859	5.928	5.981	6.022	6.056
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.74	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32



	12	15	20	25	30	40	60	100	150	∞
6.106	6.157	6.209	6.240	6.261	6.287	6.313	6.334	6.345	6.366	
99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.49	99.50	
27.05	26.87	26.69	26.58	26.50	26.41	26.32	26.24	26.20	26.13	
14.37	14.20	14.02	13.91	13.84	13.75	13.65	13.58	13.54	13.46	
9.89	9.72	9.55	9.45	9.38	9.29	9.20	9.13	9.09	9.02	
7.72	7.56	7.40	7.30	7.23	7.14	7.06	6.99	6.95	6.88	
6.47	6.31	6.16	6.06	5.99	5.91	5.82	5.75	5.72	5.65	
5.67	5.52	5.36	5.26	5.20	5.12	5.03	4.96	4.93	4.86	
5.11	4.96	4.81	4.71	4.65	4.57	4.48	4.42	4.38	4.31	
4.71	4.56	4.41	4.31	4.25	4.17	4.08	4.01	3.98	3.91	
4.40	4.25	4.10	4.01	3.94	3.86	3.78	3.71	3.67	3.60	
4.16	4.01	3.86	3.76	3.70	3.62	3.54	3.47	3.43	3.36	
3.96	3.82	3.66	3.57	3.51	3.43	3.34	3.27	3.24	3.17	
3.80	3.66	3.51	3.41	3.35	3.27	3.18	3.11	3.08	3.00	
3.67	3.52	3.37	3.28	3.21	3.13	3.05	2.98	2.94	2.87	
3.55	3.41	3.26	3.16	3.10	3.02	2.93	2.86	2.83	2.75	
3.46	3.31	3.16	3.07	3.00	2.92	2.83	2.76	2.73	2.65	
3.37	3.23	3.08	2.98	2.92	2.84	2.75	2.68	2.64	2.57	
3.30	3.15	3.00	2.91	2.84	2.76	2.67	2.60	2.57	2.49	
3.23	3.09	2.94	2.84	2.78	2.69	2.61	2.54	2.50	2.42	
3.17	3.03	2.88	2.78	2.72	2.64	2.55	2.48	2.44	2.36	
3.12	2.98	2.83	2.73	2.67	2.58	2.50	2.42	2.38	2.31	
3.07	2.93	2.78	2.69	2.62	2.54	2.45	2.37	2.34	2.26	
3.03	2.89	2.74	2.64	2.58	2.49	2.40	2.33	2.29	2.21	
2.99	2.85	2.70	2.60	2.54	2.45	2.36	2.29	2.25	2.17	
2.96	2.82	2.66	2.57	2.50	2.42	2.33	2.25	2.21	2.13	
2.93	2.78	2.63	2.54	2.47	2.38	2.29	2.22	2.18	2.10	
2.90	2.75	2.60	2.51	2.44	2.35	2.26	2.19	2.15	2.06	
2.87	2.73	2.57	2.48	2.41	2.33	2.23	2.16	2.12	2.03	
2.84	2.70	2.55	2.45	2.39	2.30	2.21	2.13	2.09	2.01	
2.66	2.52	2.37	2.27	2.20	2.11	2.02	1.94	1.90	1.80	
2.50	2.35	2.20	2.10	2.03	1.94	1.84	1.75	1.70	1.60	
2.42	2.27	2.12	2.01	1.94	1.85	1.75	1.65	1.61	1.49	
2.37	2.22	2.07	1.97	1.89	1.80	1.69	1.60	1.55	1.43	
2.31	2.16	2.00	1.90	1.83	1.73	1.62	1.52	1.46	1.33	
2.18	2.04	1.88	1.77	1.70	1.59	1.47	1.36	1.29	1.00	